Gold #2 - Solutions

I. ①-④ use defn of $f'(x)$; must identify $f(x)$ in each, then take derivative.

① $f(x) = 5x^2$, $f'(x) = 10x$ $\boxed{B}$

② $f(x) = 2\sqrt{x}$, $f'(x) = \frac{1}{\sqrt{x}}$ $\boxed{C}$

③ $f(x) = \sin x$, $f'(x) = \cos x$, $f'(\frac{\pi}{2}) = 0$ $\boxed{E}$

④ $f(x) = \cos x$, $f'(x) = -\sin x$, $f'(\frac{\pi}{2}) = -1$ $\boxed{A}$

⑤ $\sin(\theta + \frac{3\pi}{2}) = \sin\theta \cos\frac{3\pi}{2} + \cos\theta \sin\frac{3\pi}{2}$

$= \sin\theta \cdot 0 + \cos\theta(-1)$

$= -\cos\theta$ $\boxed{E}$

⑥ $g(x) = \frac{-x - f(x)}{f(x)}$, $g'(x) = \frac{f(x)[-1 - f'(x)] - f'(x)[-x - f(x)]}{[f(x)]^2}$

$$g'(1) = \frac{4(-1-2) - 2(-1-4)}{4^2}$$

$$= \frac{4(-3) - 2(-5)}{16}$$

$$= \frac{-12 + 10}{16}$$

$$= -\frac{1}{8} \quad \boxed{E}$$

(7) $s(t) = 2t^3 - 4t^2 + 2t - 1$

$v(t) = 6t^2 - 8t + 2$

$a(t) = 12t - 8$

$a(2) = 24 - 8 = 16$ $\boxed{B}$

(8) $f(x) = (5-3x)^3$

$f'(x) = 3(5-3x)^2(-3)$

$= -9(5-3x)^2$

$f'(\frac{1}{3}) = -9(5-1)^2$

$= -9 \cdot 16$

$= -144$ $\boxed{A}$

(9) $f(x) = 4x^{1/2}$

$f'(x) = 2x^{-1/2}$

$f''(x) = -x^{-3/2}$

$f'''(x) = \frac{3}{2} x^{-5/2}$

$f'''(9) = \frac{3}{2}(3)^{-5}$

$= \frac{3}{2 \cdot 243} = \frac{1}{162}$ $\boxed{B}$

(10) $x^3 + y^2 = 4$

$3x^2 + 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{3x^2}{2y}$ $\boxed{E}$

(11) $y^3 + x^2y^2 - 3x^3 = 9$

$3y^2 \frac{dy}{dx} + 2xy^2 + 2x^2y \frac{dy}{dx} - 9x^2 = 0$

$\frac{dy}{dx} = \frac{9x^2 - 2xy^2}{3y^2 + 2x^2y}$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{9-8}{12+4} = \frac{1}{16}$ $\boxed{A}$

(12) $x^2 - y^2 = 10$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

$\frac{d^2y}{dx^2} = \frac{y - x\cdot\frac{dy}{dx}}{y^2}$

$= \frac{y - x\left(\frac{x}{y}\right)}{y^2}$

$= \frac{y^2 - x^2}{y^3}$

$= \frac{-10}{y^3}$ $\boxed{C}$

(13) $y = x^2(x^2+1)^5$

$\frac{dy}{dx} = 5x^2(x^2+1)^4 \cdot 2x + 2x(x^2+1)^5$

$= 2x(x^2+1)^4\left[5x^2 + x^2 + 1\right]$

$= 2x(x^2+1)^4(6x^2+1)$ $\boxed{D}$

(14) $y = (1-4x)^{3/4}$

$y' = \frac{3}{4}(1-4x)^{-1/4}(-4)$

$= -3(1-4x)^{-1/4}$

$y'' = \frac{3}{4}(1-4x)^{-5/4}\cdot(-4)$

$= \frac{-3}{\sqrt[4]{(1-4x)^5}}$ $\boxed{A}$

(15) $f(x) = \frac{\sqrt{x}}{x+4}$

Domain: $[0, \infty)$. No VA

$\lim_{x\to\infty} f(x) = 0$, $y = 0$ $\boxed{B}$

(16) $\dfrac{dy}{dx} = 2x+6$

$\left.\dfrac{dy}{dx}\right|_{(-2,-4)} = 2$

$m_{\perp} = -\dfrac{1}{2}$

$y+4 = -\frac{1}{2}(x+2)$

$y = -\frac{1}{2}x - 5$

$x^2 + 6x + 4 = -\frac{1}{2}x - 5$

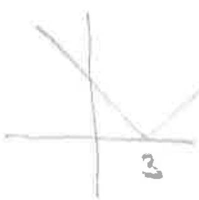
$2x^2 + 13x + 18 = 0$

$(2x+9)(x+2)$

$x = -\frac{9}{2}, \quad x = -2$

$\boxed{B}$

(17) $f(x) = |x-3|$



$\boxed{C}$

(18) $\boxed{A}$

(19) $\begin{cases} x = t^2 + 1 \\ y = t^3 \end{cases}$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \dfrac{3t^2}{2t}$

$= \frac{3}{2}t$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$

$= \dfrac{\frac{dy'}{dt}}{\frac{dx}{dt}}$

$= \dfrac{\frac{3}{2}}{2t}$

$= \frac{3}{4}t$ $\boxed{A}$

(20) $x(t) = \sin(2t) - \cos(3t)$

$v(t) = 2\cos(2t) + 3\sin(3t)$

$a(t) = -4\sin(2t) + 9\cos(3t)$

$a(\pi) = -4(0) + 9(-1) = -9$ $\boxed{E}$

(21) $f(x) = e^{\tan^2 x}$

$f'(x) = e^{\tan^2 x} \cdot 2\tan x \cdot \sec^2 x$ $\boxed{D}$

(22) $\frac{d}{dx}\left[\ln\left(\frac{1}{1-x}\right)\right] = \frac{d}{dx}\left[\ln(1-x)^{-1}\right]$

$= (1-x) \cdot \left[-(1-x)^{-2}\right] \cdot -1$

$= \frac{1-x}{(1-x)^2} = \frac{1}{1-x}$ $\boxed{A}$

or $\frac{d}{dx}\left[\ln\left(\frac{1}{1-x}\right)\right] = \frac{d}{dx}\left[\ln 1 - \ln(1-x)\right]$

$= 0 - \frac{-1}{1-x} = \frac{1}{1-x}$ $\boxed{A}$

(23) $\boxed{C}$

(24) $f(x) = x \cdot \ln(x^2)$

$f'(x) = x \cdot \frac{1}{x^2} \cdot 2x + \ln(x^2)$

$= 2 + \ln(x^2)$ $\boxed{B}$

(25) $x(t) = t \cdot e^{-2t}$

$v(t) = t \cdot e^{-2t}(-2) + e^{-2t}$

$= e^{-2t}(-2t+1)$

$v(t) = 0$ when $t = \frac{1}{2}$

$\boxed{C}$

(26) $y = x^3 + 3x^2 + 7x - 1$

$\frac{dy}{dx} = 3x^2 + 6x + 7$

$\left.\frac{dy}{dx}\right|_{x=-1} = 4$

$m_\perp = -\frac{1}{4}$

Point $(-1, -6)$

$y + 6 = -\frac{1}{4}(x+1)$

$4y + 24 = -x - 1$

$x + 4y = -25$ $\boxed{E}$

(27) $\frac{d}{dx} f(h(x)) = f'(h(x)) \cdot h'(x)$

$= g(x^2) \cdot 2x$

$= 2x \cdot g(x^2)$ $\boxed{D}$

(28) $y = \sin^{-1}(x) - (1-x^2)^{1/2}$

$y' = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot (-2x) = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$

$= \frac{1+x}{\sqrt{1-x^2}}$ $\boxed{C}$

(29) $y = x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\ln y = (\ln x)^2$

$\frac{1}{y}\frac{dy}{dx} = 2 \cdot \ln x \cdot \frac{1}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x} \cdot y$ $\boxed{C}$

(30) $\boxed{D}$

(31) $\boxed{E}$

(32) $v_1(t) = \cos t$

$v_2(t) = -2e^{-2t}$

$\cos t = -2e^{-2t}$

$\cos t + 2e^{-2t} = 0$

look at zeros

$\boxed{D}$

Remember $0 \le t \le 10$

(33) $h'(2) = \frac{1}{f'(1)}$ $\quad f'(x) = 3x^2+1$

$f'(1) = 4$

$= \frac{1}{4}$ $\boxed{B}$

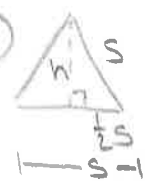
(34) $\frac{dy}{dx} = 2\cos\left(\frac{2x}{3}\right) - 3\sin\left(\frac{3x}{2}\right)$

$\left.\frac{dy}{dx}\right|_{x=\pi} = 2\left(-\frac{1}{2}\right) - 3(-1)$

$= -1 + 3$

$= 2$ $\boxed{A}$

(35)



$h^2 + \left(\frac{1}{2}s\right)^2 = s^2$

$h^2 + \frac{1}{4}s^2 = s^2$

$h = \frac{\sqrt{3}}{2}s$

$A = \frac{1}{2}b \cdot h = \frac{1}{2} \cdot s \cdot \frac{\sqrt{3}}{2}s$

$A = \frac{\sqrt{3}}{4}s^2$

$\frac{dA}{ds} = \frac{\sqrt{3}}{2}s$

$\left.\frac{dA}{ds}\right|_{s=2} = \sqrt{3}$ $\boxed{B}$

Free Response

(36) $ARoc = \dfrac{f(\frac{\pi}{2}) - f(0)}{\frac{\pi}{2} - 0}$

$= \dfrac{2-1}{\frac{\pi}{2}}$

$= \dfrac{2}{\pi}$

(37) $\dfrac{dy}{dx} = 12x$

$\left.\dfrac{dy}{dx}\right|_{x=a} = 12a$

(38) $v(t) = 2t^3 - 9t^2 + 12t - 5$

$a(t) = 6t^2 - 18t + 12$

$= 6(t^2 - 3t + 2)$

$= 6(t-2)(t-1)$

$t = 2, \quad t = 1$

$|v(1)| = 0$

$|v(2)| = 1$

(39) $v(t) = t \cdot \cos t, \quad t \geq 0$

a) moving up, $v(t) > 0$

	$(0, \frac{\pi}{2})$	$(\frac{\pi}{2}, \frac{3\pi}{2})$	$(\frac{3\pi}{2}, 5)$
t	+	+	+
$\cos t$	+	-	+
	(+)	(-)	(+)

moving up: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 5)$

$a(t) = \cos t - t \sin t$

(40) $x^3 + y^3 = 1$

a) $3x^2 + 3y^2 \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{-x^2}{y^2}$

b) $\dfrac{d^2y}{dx^2} = \dfrac{-2x \cdot y^2 + x^2 \cdot 2y \frac{dy}{dx}}{y^4}$

$= \dfrac{-2xy^2 + 2x^2\cancel{y}\left(-\frac{x^2}{y^{\cancel{2}}}\right)}{y^4}$

$= \dfrac{-2xy^2 - \frac{2x^4}{y}}{y^4}$

$= \dfrac{\frac{-2xy^3 - 2x^4}{y}}{y^4}$

$= \dfrac{-2xy^3 - 2x^4}{y^5} \rightarrow = \dfrac{-2x(x^3 + y^3)}{y^5}$

$= \dfrac{-2x}{y^5}$